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# A SPATIAL PROBLEM FOR AN ELASTIC WEDGE WITH A STRIP CUT†

D. A. POZHARSKII

Rostov-on-Don

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Integral equations are obtained for problems of the elastic equilibrium of a spatial wedge, weakened by a planar strip cut located in the median half-plane of the wedge, for different boundary conditions on the faces of the wedge. A known normal load, which is symmetrical about the plane of the cut is applied to the edges of the cut. In the case when the cut reaches the edge of the wedge, the solution of the problem using the method of paired integral equations reduces to Fredholm integral equations of the second kind with symmetric kernels. The possibility of the cleavage of the wedge along an edge [1] is shown. A simple formula which is convenient for applications is found for the normal stress intensity factor at one end of the cut. Calculations for various wedge aperture angles are carried out using this formula.

The problem of a cut in the form of a strip in a homogeneous infinite space or an infinite space which is deformed according to a power law has previously been studied [2]. Problems of cracks in a planar wedge have been treated using a Mellin integral transformation [1, 3]. A Fourier–Kontorovich–Lebedev integral transformation in the complex plane has been employed in the case of a spatial wedge [4–7].

**1.** Consider a three-dimensional elastic wedge with an aperture angle $2\alpha$ $(0<\alpha\leqslant\pi)$ in a cylindrical system of coordinates $r$, $\varphi$, $z$ with the $z$-axis directed along an edge of the wedge. There is a strip-shaped cut in the median half-plane of the wedge $\varphi=0$ which occupies a domain $\Omega$: $\{0\leqslant r\leqslant b,\ |z|<\infty\}$. The cut exists in an expanded state under the action of a load $\sigma_\varphi=-q(r,z)$, $\varphi=\pm 0$, $(r,z)\in\Omega$ which is periodic with respect to $z$ with a period of $2l$. It is assumed that the faces of the wedge are either free from stresses (problem (a)), lie in a frictionless state on a rigid base (a slipping fixing, problem (b)) or are rigidly clamped (problem (c)). It is required to find the shape of the expansion of the cut $u_\varphi=f(r,z)$ $\varphi=\pm 0$, $(r,z)\in\Omega$, whereupon it is possible to determine the normal stress intensity factor. We shall subsequently study only the domain $0\leqslant\varphi\leqslant\alpha$, as the problem is symmetrical with respect to $\varphi$. The boundary conditions in this domain can be written in the form

$$\begin{aligned}&\varphi=0:\quad \tau_{r\varphi}=\tau_{\varphi z}=0;\quad u_\varphi=0\quad (r,z)\notin\Omega\quad \sigma_\varphi=-q(r,z)\quad (r,z)\in\Omega\\&\varphi=\alpha:\quad \text{(a)}\ \sigma_\varphi=\tau_{r\varphi}=\tau_{\varphi z}=0,\quad \text{(b)}\ u_\varphi=\tau_{r\varphi}=\tau_{\varphi z}=0,\quad \text{(c)}\ u_r=u_\varphi=u_z=0\end{aligned}\tag{1.1}$$

For simplicity, we shall assume that the function $q(r,z)$ is even with respect to $z$ and can be represented in terms of a Fourier series. It then suffices to solve the problem for the case

$$q(r,z)=q(r)\cos\beta z,\quad \beta=\pi n/l\tag{1.2}$$

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and to construct a superposition of the solutions obtained for different values of $n \geq 1$ as well as of the solution of the problem of the planar deformation of a wedge $(n=0)$ [3].

On finding the three harmonic functions in the Papkovich–Neuber representation in the form of Fourier–Kontorovich–Lebedev integrals in the complex plane [4, 5], we reduce the problem to an integral equation in $f(r)$

$$(f(r,z)=f(r)\cos\beta z)$$

$$\int_0^b f(x)K(r,x)dx = \frac{1-\nu}{G} q(r), \quad 0 \leq r \leq b \tag{1.3}$$

$$K(r,x) = \frac{2}{\pi^2 rx} \int_0^\infty \frac{u \,\mathrm{sh}\, \pi u}{\mathrm{ch}(\pi u/2)} K_{iu}(\beta r)\left[I-(1-2\nu)A_m^u\right]\left\{\frac{s\,\mathrm{ch}(\pi s/2)}{W_m(s)} K_{is}(\beta x)\right\} du \tag{1.4}$$

(a) $m=1$

$$A_1^u\{g(s)\} = \int_0^\infty L_1(u,s)g(s)ds + \frac{W_0(u)}{2W_1(u)} B_*^u\{g(s)\} +$$

$$+\mathrm{ch}\frac{\pi u}{2} \int_0^\infty\int_0^\infty \frac{\mathrm{sh}\,\pi y g_1(y)\mathrm{sh}(\pi s/2)W_0(s)g(s)}{(\mathrm{ch}\,\pi y + \mathrm{ch}\,\pi u)(\mathrm{ch}\,\pi y + \mathrm{ch}\,\pi s)} dsdy +$$

$$+(1-2\nu)\mathrm{ch}\frac{\pi u}{2} \int_0^\infty\int_0^\infty \frac{\mathrm{sh}\,\pi t g_1(t)\mathrm{sh}(\pi y/2)W_2(y)}{(\mathrm{ch}\,\pi t + \mathrm{ch}\,\pi u)(\mathrm{ch}\,\pi t + \mathrm{ch}\,\pi y)} B_*^y\{g(s)\}dtdy$$

$$L_1(u,s) = 2\,\mathrm{ch}\frac{\pi u}{2}\mathrm{sh}\frac{\pi s}{2} W_1(s) \int_0^\infty \frac{\mathrm{sh}\,\pi t g_2(t)dt}{(\mathrm{ch}\,\pi t + \mathrm{ch}\,\pi u)(\mathrm{ch}\,\pi t + \mathrm{ch}\,\pi s)}$$

$$B_*^u\{g(s)\} = \sum_{n=0}^\infty (1-2\nu)^n (A_2^u)^n \circ C^t\{g(s)\} \tag{1.5}$$

$$C^t\{g(s)\} = 4\,\mathrm{ch}\frac{\pi t}{2} \int_0^\infty\int_0^\infty \frac{\mathrm{sh}\,\pi y g_1(y)\mathrm{sh}(\pi s/2)W_1(s)g(s)}{(\mathrm{ch}\,\pi y + \mathrm{ch}\,\pi t)(\mathrm{ch}\,\pi y + \mathrm{ch}\,\pi s)} dsdy + \int_0^\infty L_2(t,s)g(s)\frac{W_0(s)}{W_2(s)}ds$$

$$W_{0,1}(s) = (W_+(s) \pm W_-(s))/2, \quad W_2(s) = -W_+(s)W_-(s)/W_1(s)$$

$$g_{1,2}(t) = (g_+(t) \pm g_-(t))/2$$

$$W_\pm(s) = \pm\frac{\mathrm{ch}\,\alpha s \mp \cos\alpha}{\mathrm{sh}\,\alpha s \pm s\sin\alpha}, \quad g_\pm(t) = \left\{\begin{matrix}\mathrm{cth}(\alpha t/2)\\ \mathrm{th}(\alpha t/2)\end{matrix}\right\}\frac{\sin^2\alpha}{\mathrm{ch}\,\alpha t \mp \cos 2\alpha}$$

(b) $m=2$, (c) $m=3$

$$A_{2,3}^u\{g(s)\} = \int_0^\infty L_{2,3}(u,s)g(s)ds$$

$$L_{2,3}(u,s) = 2\,\mathrm{ch}\frac{\pi u}{2}\mathrm{sh}\frac{\pi s}{2} W_{2,3}(s) \int_0^\infty \frac{\mathrm{sh}\,\pi t g_{2,3}(t)dt}{(\mathrm{ch}\,\pi t + \mathrm{ch}\,\pi u)(\mathrm{ch}\,\pi t + \mathrm{ch}\,\pi s)}$$

$$W_3(s) = \frac{\kappa\,\mathrm{sh}\,2\alpha s - s\sin 2\alpha}{\kappa\,\mathrm{ch}\,2\alpha s + s^2(1-\cos 2\alpha) + (1+\kappa^2)/2}, \quad \kappa = 3-4\nu$$

$$g_3(t) = -\frac{\sin^2 2\alpha\,\mathrm{th}\,\alpha t}{\mathrm{ch}\,2\alpha t + \cos 4\alpha} + \sin^2\alpha\{f_1(t)[2f_2(t) - tf_3(t)] - \tag{1.6}$$

$$-f_4(t)[2f_3(t) + tf_2(t)]\}/f_5(t) - 2(1-\nu)\sin\alpha\{f_1(t)(\sin 3\alpha - \sin\alpha\,\mathrm{ch}\,2\alpha t) -$$

$$-f_4(t)\cos\alpha\,\mathrm{sh}\,2\alpha t\}/f_5(t)$$

$$f_1(t) = \kappa\,\mathrm{sh}\,2\alpha t\cos 2\alpha - t\sin 2\alpha, \quad f_2(t) = \cos 2\alpha + \sin^2 2\alpha - \mathrm{ch}\,2\alpha t$$

$$f_3(t) = \sin 2\alpha\,\mathrm{th}\,\alpha t(1+\cos 2\alpha), \quad f_4(t) = \sin 2\alpha(\kappa\,\mathrm{ch}\,2\alpha t - 1)$$

$$f_5(t) = [f_1^2(t) + f_4^2(t)](\mathrm{sh}^2\alpha t + \cos^2 2\alpha)$$

Here $G$ is the shear modulus, $\nu$ is Poisson's ratio and $I$ is the identity operator. It follows from the proposition, which was proved in [5] for an integral operator $A_2^u$ that the operator series $B_*^u$ in formula (1.5) only converges in the space of continuous functions $C_M(0, \infty)$ which are bounded on the semiaxis if $\nu > 0.053$ for any angle $\alpha$.

*Lemma* 1. The kernel of integral equation (1.3) satisfies the symmetry condition: $K(r, x) = K(x, r)$.

Lemma 1 is obvious in the case of problems (b) and (c) since the function $K(r, x)$ when $m = 2, 3$, can be represented in the form

$$K(r,x)=\frac{2}{\pi^2}\int_0^\infty \operatorname{sh}\pi u K_{iu}(\beta r)K_{iu}(\beta x)\left[\frac{u^2}{rxW_m(u)}-(1-2\nu)g_m(u)\right]du$$

In the case of problem (a), the validity of Lemma 1 is established by an analysis of each term of the Neumann series which occurs in formula (1.5), permutation of the integrals and a change of the variables of integration.

In the case of problem (b), we find, when $\alpha = \pi$, that [8, pp. 786, 984, 986]

$$K(r,x)=\frac{2}{\pi^2 rx}\int_0^\infty u^2 \operatorname{ch}\pi u K_{iu}(\beta r)K_{iu}(\beta x)du=-\frac{1}{\pi}\left(\frac{d^2}{dr^2}-\beta^2\right)K_0(\beta|r-x|) \tag{1.7}$$

The kernel of (1.7) corresponds to a problem of a strip-shaped cut in an infinite space [2].

When $\alpha = \pi/2$ in the case of problem (b), on calculating the quadratures as in (1.7), we obtain the kernel of integral equation (1.3) in the form

$$K(r,x)=\frac{2}{\pi^2 rx}\int_0^\infty u^2(\operatorname{ch}\pi u-1)K_{iu}(\beta r)K_{iu}(\beta x)du=$$
$$=-\frac{1}{\pi}\left(\frac{d^2}{dr^2}-\beta^2\right)\left[K_0(\beta|r-x|)+K_0(\beta(r+x))\right] \tag{1.8}$$

which corresponds to the symmetric problem of two identical strip-shaped cuts in infinite space.

In the case of a problem concerning a strip-shaped cut perpendicular to a boundary of a stress-free half-space (problem (a) when $\alpha = \pi/2$), we obtain a kernel in the form (the series in powers of $(1 - 2\nu)$ is truncated)

$$\begin{aligned}
K(r,x)&=\frac{2}{\pi^2 rx}\int_0^\infty [u^2\operatorname{ch}\pi u-u^2-2u^4]K_{iu}(\beta r)K_{iu}(\beta x)du+\\
&+\frac{8(1-2\nu)\beta}{\pi^2}\int_0^\infty\int_0^\infty \frac{\operatorname{ch}(\pi u/2)\operatorname{ch}(\pi y/2)}{\operatorname{ch}\pi u+\operatorname{ch}\pi y}\left(\frac{u^2}{r}+\frac{y^2}{x}\right)K_{iu}(\beta r)K_{iy}(\beta x)dudy-\\
&-\frac{16(1-2\nu)^2\beta^2}{\pi^2}\int_0^\infty\int_0^\infty\int_0^\infty \frac{\operatorname{ch}(\pi u/2)\operatorname{ch}(\pi y/2)\operatorname{ch}^2(\pi t/2)}{(\operatorname{ch}\pi t+\operatorname{ch}\pi u)(\operatorname{ch}\pi t+\operatorname{ch}\pi y)}K_{iu}(\beta r)K_{iy}(\beta x)dudydt=\\
&=-\frac{1}{\pi}\left(\frac{d^2}{dr^2}-\beta^2\right)K_0(\beta|r-x|)-\frac{1}{\pi}\left[\frac{12rx}{(r+x)^2}-1\right]\times\\
&\times\left(\frac{d^2}{dr^2}-\beta^2\right)K_0(\beta(r+x))-\frac{6\beta^2 rx}{\pi(r+x)^2}K_0(\beta(r+x))+\frac{4}{\pi}(1-2\nu)\beta^2[K_0(\beta(r+x))-\\
&-\frac{\beta}{2}(r+x)(K_1(\beta(r+x))-\int_{\beta(r+x)}^\infty K_0(t)dt)]-\frac{2}{\pi}(1-2\nu)^2\beta^2\int_{\beta(r+x)}^\infty\int_t^\infty K_0(t)dtdt
\end{aligned} \tag{1.9}$$

Formulae (1.9) can be obtained from formulae (1.14) in [9].

When $\beta \to 0$, the integral equations (1.3) reduce to the integral equations of the corresponding planar problems for a wedge [3].

We also note that the solution of Eq. (1.3) must obey the condition $f(b)=0$.

**2.** The following theorem plays an important role in the derivation and solution of the integral equations (1.3)–(1.6).

*Theorem.* The operators $I-(1-2\nu)A_m^u: C_M(0,\ \infty)\to C_M(0,\ \infty)$ $(m=1,\ 2,\ 3)$, defined by formulae (1.5)–(1.6), have inverses which are equal to

$$\left[I-(1-2\nu)A_m^u\right]^{-1}=I+B_m^u,\quad B_1^u=\frac{W_+(u)}{2W_1(u)}B_+^u-\frac{W_-(u)}{2W_1(u)}B_-^u$$

$$B_{\pm,2,3}^u=\sum_{n=1}^{\infty}(1-2\nu)^n\left(A_{\pm,2,3}^u\right)^n,\quad A_\pm^u\{g(s)\}=\int_0^\infty L_\pm(u,s)g(s)ds \tag{2.1}$$

$$L_\pm(u,s)=2\operatorname{ch}\frac{\pi u}{2}\operatorname{sh}\frac{\pi s}{2}W_\pm(s)\int_0^\infty\frac{\operatorname{sh}\pi t g_\pm(t)dt}{(\operatorname{ch}\pi t+\operatorname{ch}\pi u)(\operatorname{ch}\pi t+\operatorname{ch}\pi s)}$$

when the inequality

$$(1-2\nu)\left\|A_{\pm 2,3}^u\right\|_{C_M(0,\infty)}<1 \tag{2.2}$$

is satisfied.

The proof of this theorem, which is obvious when $m=2,\ 3$ (the inversion of a Neumann series) and in the case when $m=1$ (the inversion of a combination of two Neumann series) is based on the solution of the following auxiliary problem using an integral Fourier–Kontorovich–Lebedev transformation ($\delta(x)$ is a delta-function)

$$\begin{aligned}&\varphi=\alpha/2:\quad \tau_{r\varphi}=\tau_{\varphi z}=0;\quad u_\varphi=\delta(r-x)\delta(|z|-y)\\&\varphi=-\alpha/2:\quad \sigma_\varphi=\tau_{r\varphi}=\tau_{\varphi z}=0\end{aligned} \tag{2.3}$$

as well as on the well-known connection (see [10], for example) between contact problems and problems of the crack theory. Here, we note that an operator $B_1^u$ of the form of (2.1) occurs in the kernel of integral equation (2.1) of the contact problem (a) from [7].

Investigations [5–7] have shown that inequality (2.2) is satisfied for angles $\alpha=\pi n/12$ $(n=1,2,\ldots,12)$, as a rule, for all values of $\nu$ which are encountered in practice.

To solve integral equation (1.3), we employ the method of paired [dual] integral equations [11]. Let us introduce the function

$$Q(u)=\frac{\operatorname{sh}\pi u W_m(u)}{\operatorname{ch}(\pi u/2)}\left[I-(1-2\nu)A_m^u\right]\left\{\frac{s\operatorname{ch}(\pi s/2)}{W_m(s)}\int_0^b\frac{f(x)}{x}K_{is}(\beta x)dx\right\} \tag{2.4}$$

Then, by the theorem formulated above, we can express the function from (2.4) as

$$f(x)=\frac{2}{\pi^2}\int_0^\infty\frac{\operatorname{sh}\pi u W_m(u)}{\operatorname{ch}(\pi u/2)}\left[I+B_m^u\right]\left\{\frac{Q(s)\operatorname{ch}(\pi s/2)}{\operatorname{sh}\pi s W_m(s)}\right\}K_{iu}(\beta x)du \tag{2.5}$$

Let us write a paired integral equation which is equivalent to Eq. (1.3)

$$\int_0^\infty Q(u)\frac{u}{W_m(u)}K_{iu}(\beta r)du=\frac{\pi^2}{2}\frac{1-\nu}{G}rq(r),\quad 0\leqslant r<b$$

$$\int_0^\infty \frac{\operatorname{sh}\pi u W_m(u)}{\operatorname{ch}(\pi u/2)}\left[I+B_m^u\right]\left\{\frac{Q(s)\operatorname{ch}(\pi s/2)}{\operatorname{sh}\pi s W_m(s)}\right\}K_{iu}(\beta r)du=0,\quad b\leqslant r<\infty \tag{2.6}$$

We will seek the solution of Eq. (2.6) in the form

$$\begin{aligned} &Q(u)=N(u)+M(u)\\ &N(u)=\frac{1-\nu}{G}\operatorname{sh}\pi u W_m(u)\int_0^b q(r)K_{iu}(\beta r)dr\\ &M(u)=-\left(\frac{2}{\pi}\right)^{3/2}\operatorname{sh}\pi u W_m(u)\int_b^\infty \varphi(t)\operatorname{Re}K_{1/2+iu}(\beta t)dt \end{aligned} \tag{2.7}$$

On introducing the representation (2.7) into (2.6), we satisfy the first equation of (2.6) identically [11]. The second equation of (2.6) is transformed to a Fredholm integral equation of the second kind in the function $\varphi(t)$

$$\varphi(t)+\int_b^\infty \varphi(s)R(s,t)ds=F(t),\quad b\leqslant t<\infty \tag{2.8}$$

$$\begin{aligned} R(s,t)=\frac{4\beta}{\pi^2}\int_0^\infty \operatorname{sh}\pi u\Big[&\left(W_m(u)-\operatorname{cth}\pi u\right)\operatorname{Re}K_{1/2+iu}(\beta s)+\\ &+\frac{W_m(u)}{\operatorname{ch}(\pi u/2)}B_m^u\left\{\operatorname{ch}\frac{\pi y}{2}\operatorname{Re}K_{1/2+iy}(\beta s)\right\}\operatorname{Re}K_{1/2+iu}(\beta t)du \end{aligned} \tag{2.9}$$

$$F(t)=\sqrt{\frac{2}{\pi}}\,\beta\frac{1-\nu}{G}\int_0^\infty \frac{\operatorname{sh}\pi u W_m(u)}{\operatorname{ch}(\pi u/2)}\left[I+B_m^u\right]\left\{\operatorname{ch}\frac{\pi y}{2}\int_0^b q(x)K_{iy}(\beta x)dx\right\}\operatorname{Re}K_{1/2+iu}(\beta t)du$$

*Lemma* 2. The kernel of integral equation (2.8) of the form of (2.9) is symmetrical, that is, $R(s,t)=R(t,s)$.

This lemma, the proof of which is analogous to the proof of Lemma 1, enables one to use the Hilbert–Schmidt theory [12] to investigate Eq. (2.8).

It can be shown that the condition $f(b)=0$ is satisfied in the case of a function $f(x)$ of the form of (2.5) and (2.7).

It follows from formulae (2.5) and (2.7) and the fact that $K_{iu}(0)=\pi\delta(u)$ that the behaviour of the function $f(x)$ on the edge of a wedge $(x\to 0)$ depends on the value of the limit $\lim \operatorname{sh}\pi u W_m(u)=A$ $(u\to 0,\ m=1,2,3)$. In the case of problem (b) when $\alpha=\pi$ (an infinite space) and problem (c) for any $\alpha$, we have that $A=f(0)=0$. In the remaining cases $A\neq 0$, $f(0)\neq 0$, that is, there is a cleavage of the elastic wedge along an edge.

The method of mechanical quadratures using the Gaussian quadrature formula is effective in the numerical solution of Eq. (2.8). Tables of the functions $K_{1/2+iu}(x)$ which are available [13] facilitate its use. When the functions $R(s,t)$ and $F(t)$ are calculated using formulae (2.9) instead of summation of the Neumann series for $B_m^u$ it is more convenient to solve the corresponding Fredholm integral equations of the second kind by the method of mechanical quadratures.

The problem has an exact solution in case (a) for $\alpha=\pi$ when $R(s,t)\equiv 0$.

The normal stress intensity factor when $r=b$ referred to $\cos\beta z$ can be found using the formula

$$K_I=-\lim_{r\to b+0}\left[\sqrt{r-b}\,q(r)\right] \tag{2.10}$$

$$q(r)=\frac{G}{1-\nu}\int_0^b f(x)K(r,x)dx,\quad r>b \tag{2.11}$$

Separating out the smooth part of the kernel $K(r, x)$ in (2.11) in the form of (1.7), using formulae (2.5) and (2.7) and integrating by parts, we finally obtain

$$K_I = \frac{2G\varphi(b)}{\pi^2(1-\nu)\sqrt{\beta}} \tag{2.12}$$

where $\varphi(t)$ is the solution of Eq. (2.8).

In order to carry out an actual calculation, we introduce the following dimensionless quantities

$$r' = \frac{r}{b}, \quad x' = \frac{x}{b}, \quad \beta' = \beta b, \quad f'(x') = \frac{f(x)}{b}, \quad q'(r') = \frac{q(r)}{G}, \quad K_I' = \frac{K_I}{\sqrt{bG}} \quad \text{and so on} \tag{2.13}$$

The results of calculations of the quantity $K_I'/q$ using (2.12) and (2.13) for different values of $\beta'$ are given below in the case of problem (a) when $\alpha = \pi n/8$, $\nu = 0.3$, $q'(r') = q = \text{const}$

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $\beta' = 1$ | 3.20 | 1.28 | 0.747 | 0.554 | 0.519 | 0.493 | 0.482 |
| $\beta' = 2$ | 2.29 | 0.790 | 0.478 | 0.403 | 0.395 | 0.387 | 0.383 |
| $\beta' = 3$ | 1.37 | 0.481 | 0.352 | 0.327 | 0.325 | 0.323 | 0.322 |

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